North Sydney Boys High School

MATHEMATICS

2015 HSC Course Assessment Task 3 (Trial Examination)
June 18, 2015

General instructions

- Working time – 3 hours. (plus 5 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- Attempt **all** questions.
- At the conclusion of the examination, bundle the booklets + answer sheet used in the correct order within this paper and hand to examination supervisors.

SECTION I

- Mark your answers on the answer sheet provided (numbered as page 13)

SECTION II

- Commence each new question on a new page. Write on both sides of the paper.
- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

STUDENT NUMBER: .... **# BOOKLETS USED**: .....

Class (please ✔)

◯ 12M2A – Miss Lee

◯ 12M3A – Mrs Ziaziaris

◯ 12M3B – Mr Berry

◯ 12M3C – Mr Zuber

Marker's use only.

QUESTION	1-10	11	12	13	14	15	16	**Total**
MARKS	/10	/15	/15	/15	/15	/15	/15	/100

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer sheet provided.

Questions **Marks**

1. Solving the equation $2^{2x} - 5(2^x) + 4 = 0$ gives 2 solutions for x. Which pair of solutions is correct? **1**

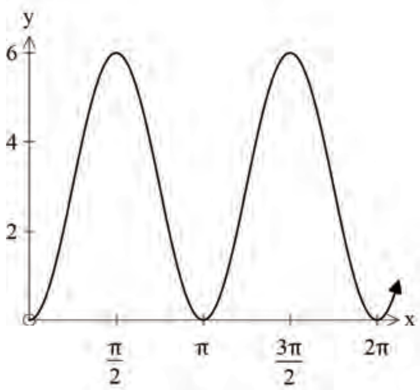
(A) $x = 1$ or $x = 0$

(B) $x = \log_2 2$ or $x = \log_2 1$

(C) $x = 2$ or $x = 0$

(D) $x = 4$ or $x = 1$

2. The equation of the graph below is given by $y = A\cos Bx + 3$. **1**



Which of the following are the values of A and B?

(A) $A = 3,\ B = 2$

(B) $A = -3,\ B = 2$

(C) $A = 6,\ B = 3$

(D) $A = -6,\ B = \pi$

3. A parabola has its focus at $(0, 2)$. The equation of its directrix is $x = -2$. Which of the following is the equation of the parabola? **1**

(A) $(x + 1)^2 = 4(y - 2)$

(B) $x^2 = 8y$

(C) $(y + 1)^2 = 4(x - 2)$

(D) $y - 2)^2 = 4(x + 1)$

4. The value of $\log_3 5000$ is closest to: **1**

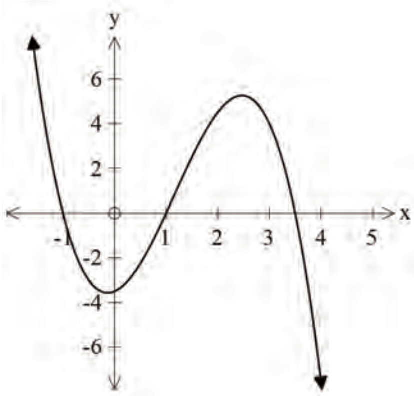
(A) 1.5

(B) 7.8

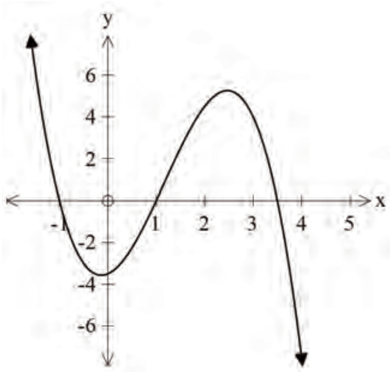
(C) 2.2

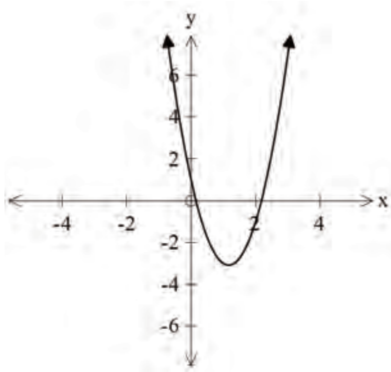
(D) 5.7

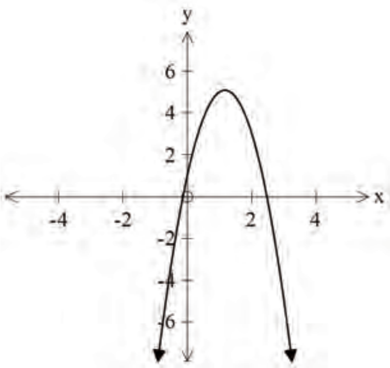
5. Examine the graph of $f(x)$ supplied. **1**

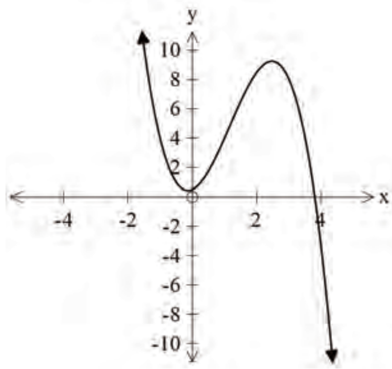


Which of the graphs below best represents $f'(x)$?

(A) 

(C) 

(B) 

(D) 

6. A circle has the equation $x^2 - 8x + y^2 - 1 = 0$. It has a radius of: **1**

(A) 17

(C) 1

(B) 4

(D) $\sqrt{17}$

7. If $\sin\theta = \dfrac{5}{13}$ and $\cos\theta < 0$, what is the exact value of $\tan\theta$? **1**

(A) $\dfrac{5}{12}$

(C) $-\dfrac{5}{12}$

(B) $\dfrac{12}{5}$

(D) $-\dfrac{12}{5}$

8. Suppose that the point $P\,(a, f(x))$ lies on the curve $y = f(x)$. **1**
If $f'(a) = 0$ and $f''(a) > 0$, which of the following statements describes the point P on the graph of $y = f(x)$?

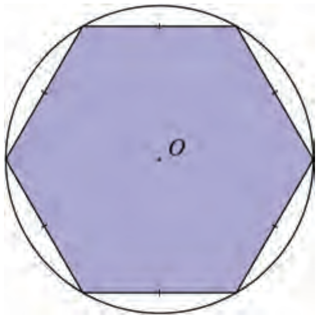
(A) P is a maximum turning point

(B) P is a minimum turning point

(C) P is a stationary point of inflexion

(D) P is a point of inflexion

9. A regular hexagon is cut from a circle with centre O, such that each vertex of the hexagon lies on the circumference of the circle. **1**



What percentage of the circle (to the nearest whole number) is the area of the hexagon?

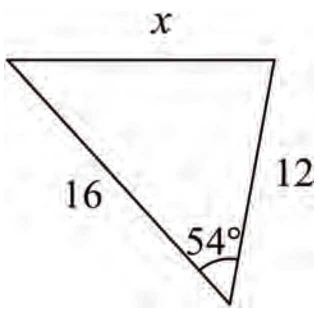
(A) 85%

(B) 84%

(C) 83%

(D) 82%

10. The value of x in the diagram to the nearest whole number is: **1**



(A) 13

(B) 153

(C) 25

(D) 10

Examination continues overleaf...

Section II

90 marks
Attempt Questions 11 to 16
Allow approximately 2 hours and 45 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available. Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks) Commence a NEW page. **Marks**

(a) Solve $|2x - 1| < 5$ **2**

(b) Fully factorise $2x^3 - 54$ **2**

(c) Simplify fully $\dfrac{1 - \cos^2\theta}{\sin\theta\cos\theta}$ **2**

(d) State the domain of $f(x) = \sqrt{81 - x^2}$ **1**

(e) Rationalise the denominator and simplify $\dfrac{1 - \sqrt{2}}{3 + \sqrt{2}}$ **2**

(f) The first term of an arithmetic series is 3, and the ninth term is five times the second term. Find the common difference. **2**

(g) Differentiate the following with respect to x. Simplify where possible.

i. $\sqrt{1 - 2x}$ **2**

ii. $\dfrac{e^{2x}}{x^2}$ **2**

Question 12 (15 Marks) Commence a NEW page. **Marks**

(a) Evaluate the following integrals:

i. $\displaystyle\int \left(\sec^2 x + 3\cos x\right)\, dx$ **2**

ii. $\displaystyle\int 2^x\, dx$ **2**

(b) Evaluate $\displaystyle\int_2^7 \frac{x}{x^2-1}\, dx$, leaving your answer in the simplest exact form. **3**

(c) For the equation $2x^2 + 3x - 7 = 0$, evaluate:

i. $\alpha + \beta$ **1**

ii. $\alpha\beta$ **1**

iii. $(\alpha - 2)(\beta - 2)$ **2**

(d) A function is defined by $f(x) = 2x^3 - 6x + 3$.

i. Find the coordinates of the turning points of the graph $y = f(x)$ and determine their nature. **2**

ii. Hence sketch the graph of $y = f(x)$, showing the turning points and the y intercept. **2**

Question 13 (15 Marks) Commence a NEW page. **Marks**

(a) The points A and B have coordinates $(1, 0)$ and $(7, 4)$ respectively. The angle between the line AB and the x-axis is θ.

i. Find the gradient of the line AB. **1**

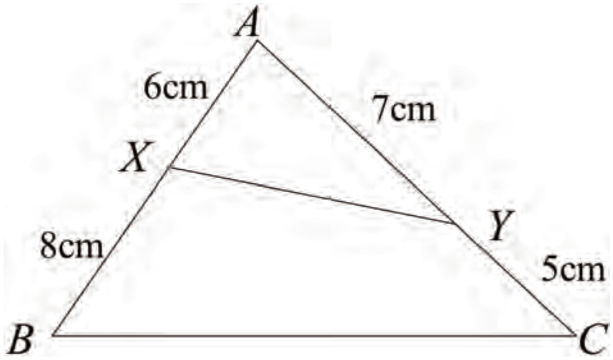
ii. Calculate the size of angle θ in degrees. **1**

iii. Find the length of interval AB. **1**

iv. Find the equation of the line AB **2**

v. Find the coordiantes of C, the midpoint of AB **1**

(b) Copy the diagram below into your examination booklet.



i. Prove $\triangle ABC \parallel\!| \triangle AXY$ **2**

ii. If $XY = 9\,\text{cm}$, find the length of BC. **1**

(c) For what value(s) of k does the equation below have no real roots? **3**

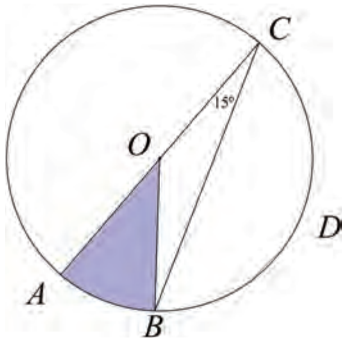
$$4x^2 + (k+3)x + 1 = 0$$

(d) Using Simpson's rule with five function values, find an estimate for the definite integral, correct to 2 decimal places: **3**

$$\int_2^6 \log_e(x-1)\,dx$$

Question 14 (15 Marks) Commence a NEW page. **Marks**

(a) In the figure, O is the centre of the circle. The length of minor arc $AB = 5\text{cm}$, $\angle ACB = 15°$.



Find in terms of π:

i. the length of radius OA **2**

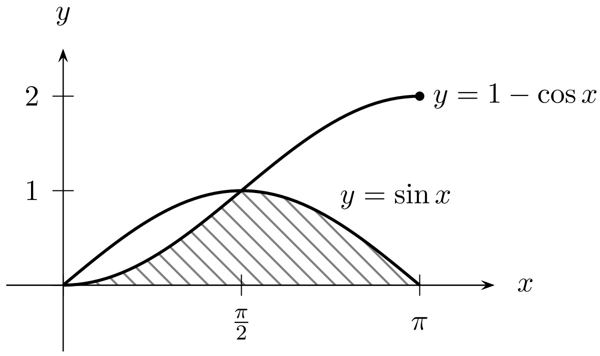
ii. the area of the shaded sector AOB **1**

iii. the area of the minor segment BDC **2**

(b) The diagram shows the graphs of the functions $y = 1 - \cos x$ and $y = \sin x$ between $x = 0$ and $x = \pi$. **4**

The graphs intersect at $x = \dfrac{\pi}{2}$.

Find the area of the shaded region.



(c) For the geometric series

$$1 - 3x + 9x^2 - \cdots$$

i. find the values of x for which the limiting sum exists. **2**

ii. find the value of x for which the limiting sum is $\dfrac{4}{5}$ **2**

(d) Shade the region which satisfies all the inequalities: **2**

$$\begin{cases} x \geq 0 \\ y \geq x^2 \\ y \leq \sqrt{9 - x^2} \end{cases}$$

Question 15 (15 Marks) Commence a NEW page. **Marks**

(a) Evaluate $\displaystyle\sum_{k=2}^{5} \frac{k^2}{k+1}$ **2**

(b) Find the equation of the locus of the point $P(x, y)$ which moves so that it is equidistant from $A(-1, 2)$ and $B(5, -3)$. **2**

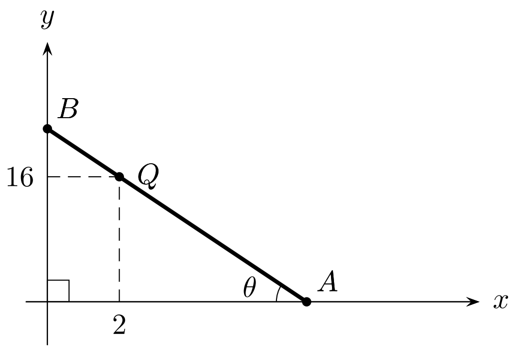
(c) R is the region bounded by the y-axis, the x-axis, the line $x = \dfrac{\pi}{2}$ and the curve $y = \sqrt{1 + \sin x}$. Show that the volume of the solid formed when R is rotated about the x-axis is **3**

$$V = \frac{1}{2}\pi\left(\pi + 2\right)$$

(d) i. Prove that the x-values of the points of intersection of the hyperbola $y = \dfrac{k}{x}$ and the line $kx + y + 2 = 0$ are given by the solution of the equation $kx^2 + 2x + k = 0$. **1**

ii. Find the values of k for which the hyperbola and the line will intersect in two distinct points. **2**

(e) A straight road is to be built from A to B. The road must pass through Q, a vertex of the rectangular block of land 16 km by 2 km as shown in the diagram below. AB makes an angle of θ with AP.

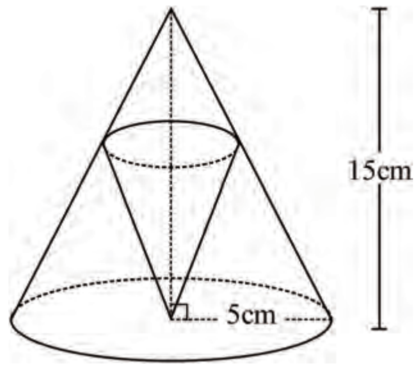


i. Show $AB = \dfrac{16}{\sin\theta} + \dfrac{2}{\cos\theta}$ **1**

ii. Show that $\tan\theta = 2$ gives the minimum distance for AB **4**

Question 16 (15 Marks) Commence a NEW page. **Marks**

(a) A small cone is enclosed within a larger cone as shown in the diagram below. The large cone has height 15cm and radius 5cm. Let h represent the height of the small cone and r represent the radius.



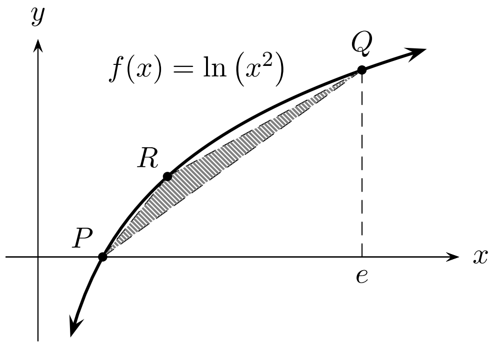
i. Show $h = 15 - 3r$ **1**

ii. Find the dimensions of the small cone for which the volume of the small cone is maximum. **3**

(b) The diagram shows the graph of the function $y = \ln(x^2), (x > 0)$.

The points $P(1, 0)$, $Q(e, 2)$ and $R(t, \ln t^2)$ all lie on the curve.

The area of $\triangle PQR$ is maximum when the tangent at R is parallel to the line through P and Q.



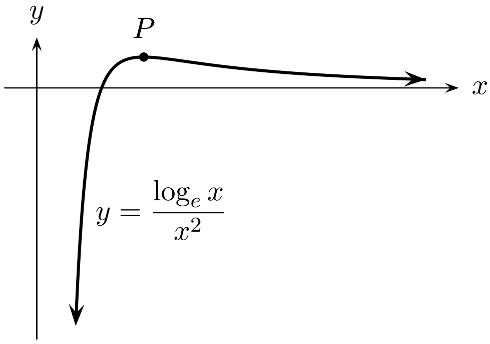
i. Find gradient of the line through P and Q. **1**

ii. Find the value of t that gives the maximum area for $\triangle PQR$. **2**

iii. Hence find the maximum area of $\triangle PQR$ **2**

Examination continues overleaf...

(c) The diagram below is the graph of $y = \dfrac{\log_e x}{x^2}$. P is a maximum turning point.



i. Find the coordinates of the point P. **3**

ii. Find the values of h such that $\dfrac{\log_e x}{x^2} = h$ has two values. **1**

iii. Find the values of k such that $\dfrac{\log_e x}{x^2} = kx$ has two solutions. **2**

End of paper.

2unit Trial 2015

1. $a^2 - 5a + 4 = 0$

$(a-4)(a-1) = 0$

$a = 4, \ a = 1$

$2^x = 4, \ 2^x = 1$

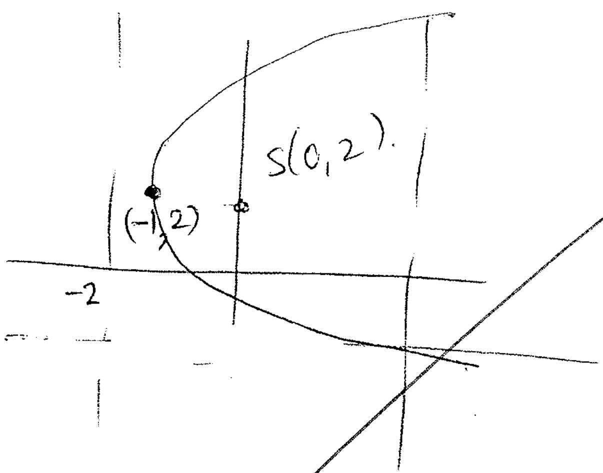
$x = 2, \ x = 0$ C

2. $\frac{2\pi}{b} = \pi$

$\therefore b = 2$

$a = -3$. B

3.



Vertex $(-1,2)$ $a = 1$

$(y-2)^2 = 4a(x+1)$

$(y-2)^2 = 4(x+1)$ D.

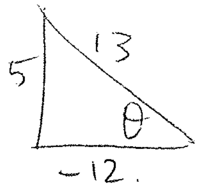
4. $\frac{\log_e 5000}{\log_e 3}$ B.

5. B

6. $x^2 - 8x + (-4)^2 + y^2 = 1 + 16$

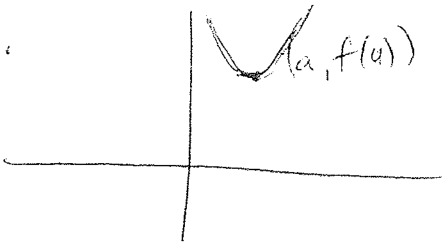
$(x-4)^2 + y^2 = 17$ D.

7.



C

8.



B.

$A_o = \pi r^2$

$A_{HEX} = \frac{1}{2} \times r^2 \sin 60° \times 6.$

$= \frac{r^2}{2} \times \frac{\sqrt{3}}{2} \times 6$

$= \frac{r^2\sqrt{3}}{4} \times 6$

$= \frac{3\sqrt{3}r^2}{2}$

$\frac{3\sqrt{3}r^2}{2} \div \pi r^2 \times 100\%$

$= \frac{3\sqrt{3}\cancel{r^2}}{2} \times \frac{1}{\pi \cancel{r^2}}$

$= \frac{3\sqrt{3}}{2\pi} \times 100\%$

$= 83\%$

C.

10. $x = \sqrt{16^2 + 12^2 - 2(16)(12)\cos 54}$

$= 13.2 \quad (1 \text{ d.p.})$

A

Section II

-1 if not in order

11. a) $|2x-1| < 5$

$-5 < 2x-1 < 5$ 1

$-4 < 2x < 6$

$\boxed{-2 < x < 3}$ 1

b) $2x^3 - 54$

$= 2(x^3-27)$ 1

$= 2(x-3)(x^2+3x+9)$ 1

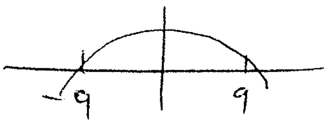
c) $\dfrac{1-\cos^2\theta}{\sin\theta\cos\theta}$ 1

$= \dfrac{\sin^2\theta}{\sin\theta\cos\theta}$

$= \tan\theta$ 1

d) $D: \ 81-x^2 \geq 0$

$(9-x)(9+x) \geq 0$



$\boxed{-9 \leq x \leq 9}$ 1

e) $\dfrac{(1-\sqrt{2})}{(3+\sqrt{2})} \times \dfrac{(3-\sqrt{2})}{(3-\sqrt{2})}$ 1

$= \dfrac{3-\sqrt{2}-3\sqrt{2}+2}{9-2}$

$= \dfrac{5-4\sqrt{2}}{7}$ 1

f) $a = 3$

$a+8d = 5(a+d)$ 1

$3+8d = 5(3+d)$

$3+8d = 15+5d$

$3d = 12$

$d = 4.$ 1

g) (i) $\dfrac{d}{dx}(1-2x)^{1/2}$

Need to simplify

$= \frac{1}{2}(-2)(1-2x)^{-1/2}$ 1

$= -\dfrac{1}{\sqrt{1-2x}}.$ 1

(ii) $\dfrac{d}{dx}\left(\dfrac{\sin 2x}{x^2}\right)$

Don't need to simplify

$= \dfrac{x^2 . 2\cos 2x - \sin 2x . 2x}{x^4}$

$= \dfrac{2x(x\cos 2x - \sin 2x)}{x^4}$

$= \dfrac{2(x\cos 2x - \sin 2x)}{x^3}$ 1

$\dfrac{d}{dx}\left(\dfrac{e^{2x}}{x^2}\right)$

$= \dfrac{2e^{2x}x^2 - 2xe^{2x}}{x^4}$ 1

$= \dfrac{2xe^{2x}(x-1)}{x^4}$ 1

$= \dfrac{2e^{2x}(x-1)}{x^3}$ 1

12. a) (i) $\int (\sec^2 x + 3\cos x)\, dx$

$= \tan x + 3\sin x + c$ 2

(ii) $\int 2^x \, dx$

$= \int e^{\ln 2^x} \, dx$

$= \int e^{x\ln 2} \, dx$

$= \dfrac{e^{x\ln 2}}{\ln 2}$ 1

$= \dfrac{2^x}{\ln 2} + c$ 1

b) $\int_2^7 \dfrac{x}{x^2-1} \, dx$

$= \dfrac{1}{2}\int_2^7 \dfrac{2x}{x^2-1} \, dx$

$= \dfrac{1}{2}\left[\ln|x^2-1|\right]_2^7$ 1

$= \dfrac{1}{2}\left[\ln 48 - \ln 3\right]$

$= \dfrac{1}{2}\ln\left(\dfrac{48}{3}\right)$ 1

$= \dfrac{1}{2}\ln 16$

$= \ln 16^{1/2}$

$= \ln 4.$ 1

c) (i) $2x^2 + 3x - 7 = 0$

$\alpha + \beta = -\dfrac{3}{2}$ 1

(ii) $\alpha\beta = \dfrac{-7}{2}$ 1

(iii) $(\alpha-2)(\beta-2)$

$= \alpha\beta - 2\alpha - 2\beta + 4$

$= \alpha\beta - 2(\alpha+\beta) + 4$ 1

$= -\dfrac{7}{2} - 2\left(-\dfrac{3}{2}\right) + 4$

$= -\dfrac{7}{2} + 3 + 4$

$= 3\dfrac{1}{2}.$ 1

d) $f(x) = 2x^3 - 6x + 3$

(i) $f'(x) = 6x^2 - 6$

$f''(x) = 12x$

Stat Pts: $f'(x) = 0$

$6x^2 - 6 = 0$

$6x^2 = 6$

$x^2 = 1$

$x = 1, -1$

$y = -1, 7$

Test $(1,-1)$ 1

$f''(x) = 12$

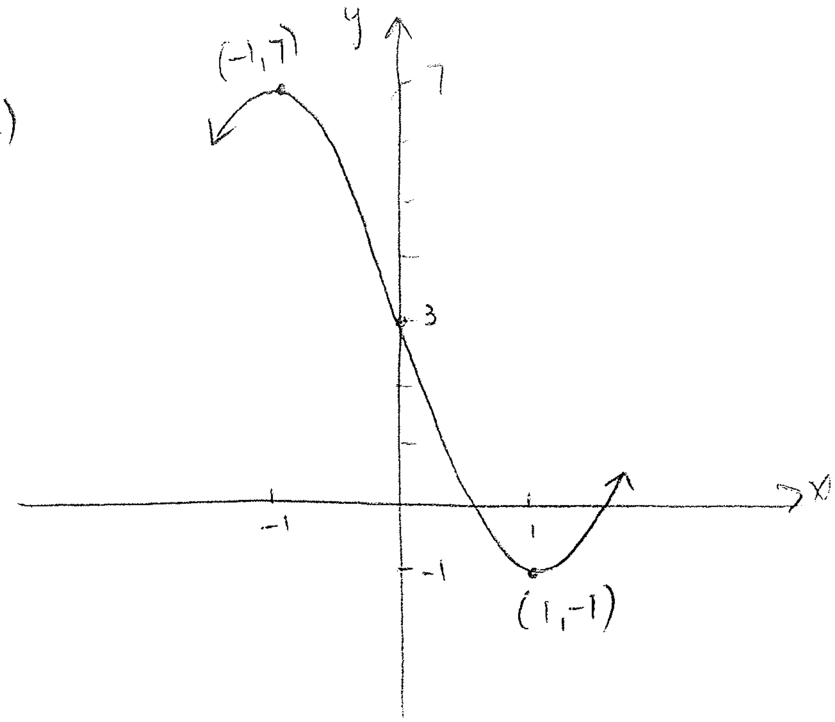
$> 0 \quad \therefore \boxed{\min (1,-1)}$

Test $(-1,7)$

$f''(x) = -12$ 1

$< 0 \quad \therefore \boxed{\max -1,7}$

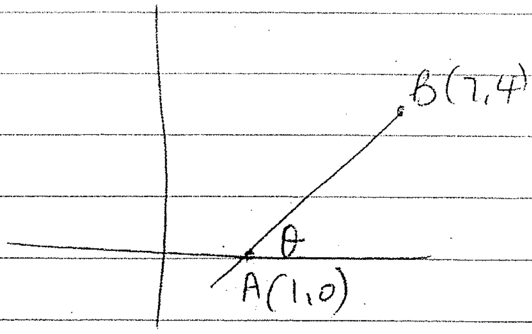
(ii)



1 y-intercept

1 turning pts

13 a) $A(1,0)$ $B(7,4)$



(i) $m = \frac{4}{6} = \frac{2}{3}$ 1

(ii) $\tan\theta = \frac{2}{3}$

$\theta = 33°41'$ 1

(iii) $d = \sqrt{(7-1)^2 + (4-0)^2}$

$= \sqrt{36+16}$

$= \sqrt{52}$ units 1

$= 7.2$ units (1dp)

(iv) $y - 0 = \frac{2}{3}(x-1)$ 1

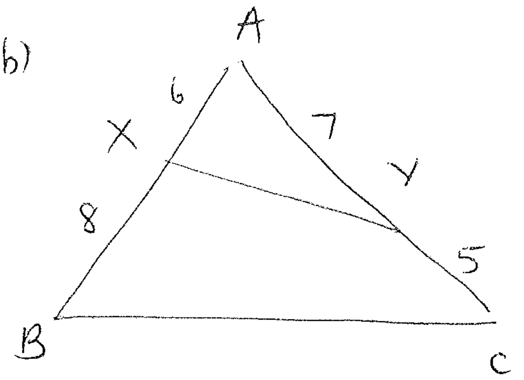
$3y = 2x - 2$

$2x - 3y - 2 = 0$ 1

(v) $C(x,y) = \left(\frac{1+7}{2}, \frac{4+0}{2}\right)$

$= (4, 2)$ 1

13. b)



(i) In Δ's AXY & ACB

1. $\angle A$ is common

2. $\frac{AY}{AB} = \frac{7}{14} = \frac{1}{2}$

$\frac{AX}{AC} = \frac{6}{12} = \frac{1}{2}$ 1

$\therefore \Delta AXY \,|||\, \Delta ACB$

(two sides in proportion & included angle equal) 1

(ii) BC = 18 cm

(corresponding sides of similar triangles in same ratio) 1

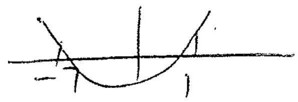
c) No real roots : $b^2 - 4ac < 0$

$(k+3)^2 - 4(4)(1) < 0$ 1

$k^2 + 6k + 9 - 16 < 0$

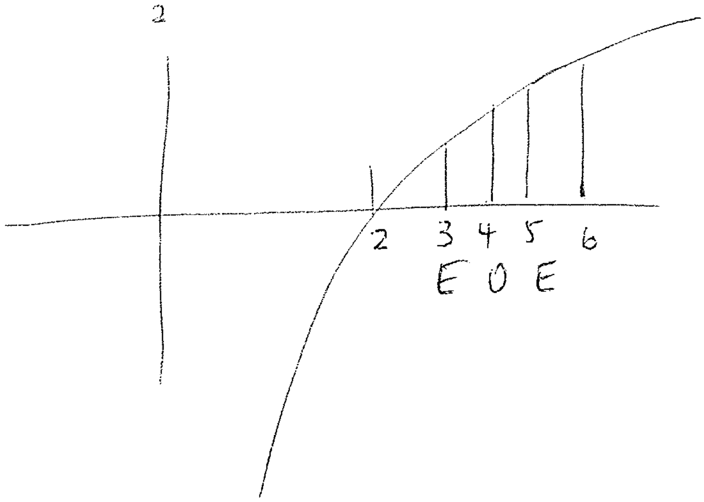
$k^2 + 6k - 7 < 0$

$(k+7)(k-1) < 0$ 1



$\boxed{-7 < k < 1}$ 1

d) $\int_2^6 \log_e(x-1)\, dx$



$$A \doteqdot \frac{1}{3}\left\{f(2) + 4\left[f(3) + f(5)\right] + 2\left[f(4)\right] + f(6)\right\}$$ 1

$$= \frac{1}{3}\left\{\ln 1 + 4(\ln 2 + \ln 4) + 2\ln 3 + \ln 5\right\}$$ 1

$$= 4.04$$ 1

14\. a) (i) $\angle BOC = 15^\circ$ (isos. Δ)

$\therefore \angle AOB = 30^\circ$ (ext. angle of a triangle rule)

$= \frac{\pi}{6}$

$l = r\theta$

$5 = r \times \frac{\pi}{6}$ 1

$r = \frac{30}{\pi}$ cm. 1

(ii) $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times \left(\frac{30}{\pi}\right)^2 \times \frac{\pi}{6}$

$= \frac{900 \cancel{\pi}}{12 \pi^{\cancel{2}}}$

$= \frac{75}{\pi}$ cm^2. 1

(iii) $\angle COB = 150^\circ = 150 \times \frac{\pi}{180}$

$= \frac{5\pi}{6}$

$A = \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} \times \left(\frac{30}{\pi}\right)^2 \left(\frac{5\pi}{6} - \sin\frac{5\pi}{6}\right)$ 1

$= \frac{1}{2} \times \frac{900}{\pi^2} \left(\frac{5\pi}{6} - \frac{1}{2}\right)$

$= \frac{450}{\pi^2} \times \frac{5\pi}{6} - \frac{225}{\pi^2}$

$= \frac{375}{\pi} - \frac{225}{\pi^2}$

$= \frac{375\pi - 225}{\pi^2}$

$= 96.57$ cm^2 (2dp) 1

b) $A = \int_0^{\pi/2} (1-\cos x)\,dx + \int_{\pi/2}^{\pi} \sin x \, dx$ 1

$= \left[x - \sin x\right]_0^{\pi/2} + \left[-\cos x\right]_{\pi/2}^{\pi}$ 1

$= \frac{\pi}{2} - \sin\frac{\pi}{2} + \left[-\cos\pi + \cos\frac{\pi}{2}\right]$ 1

$= \frac{\pi}{2} - 1 + 1 + 0$

$= \frac{\pi}{2}\ u^2$ 1

c) (i) Limiting Sum exists when

$-1 < r < 1$

$-1 < -3x < 1$ 1

$1 > \cancel{3x} > -\frac{1}{3}$

$\therefore \boxed{-\frac{1}{3} < x < \frac{1}{3}}$ 1

(ii) $S_\infty = \frac{a}{1-r}$.

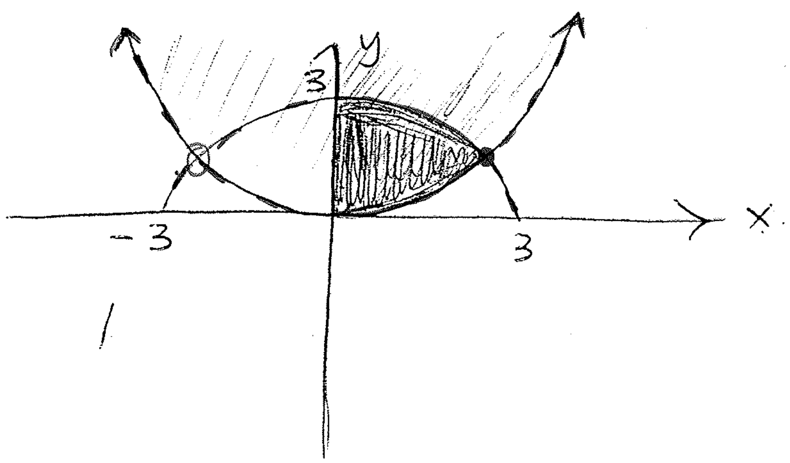
$\frac{4}{5} = \frac{1}{1+3x}$ 1

$4 + 12x = 5$

$12x = 1$

$x = \frac{1}{12}$ 1

d)



1 curves

1 region

15. (a) $\displaystyle\sum_{k=2}^{5} \frac{k^2}{k+1}$

$k=2: \quad \dfrac{2^2}{3} = \dfrac{4}{3}$

$k=3: \quad \dfrac{3^2}{4} = \dfrac{9}{4}$

$k=4: \quad \dfrac{4^2}{5} = \dfrac{16}{5}$

$k=5: \quad \dfrac{5^2}{6} = \dfrac{25}{6}$ 1

Sum $= \dfrac{219}{20}$ 1

(b) $(x+1)^2 + (y-2)^2 = (x-5)^2 + (y+3)^2$ 1

$x^2 + 2x + y^2 - 4y + 5 = x^2 - 10x + y^2 + 6y + 34$

$12x - 10y - 29 = 0$ 1

c) $V = \pi \displaystyle\int_0^{\pi/2} (1 + \sin x)\, dx$ 1

$= \pi \left[x - \cos x \right]_0^{\pi/2}$ 1

$= \pi \left[\dfrac{\pi}{2} - \cos\dfrac{\pi}{2} - 0 + 1 \right]$

$= \pi \left[\dfrac{\pi}{2} + 1 \right]$

$V = \dfrac{1}{2}\pi \left[\pi + 2 \right]$ 1

15 d) (i) Solve simultaneously

$$y = \frac{k}{x} \quad —①$$

$$kx + y + 2 = 0 \quad —②$$

Sub ① into ②

$$kx + \frac{k}{x} + 2 = 0$$

$$kx^2 + k + 2x = 0$$

(ii) 2 distinct points $\Delta > 0$.

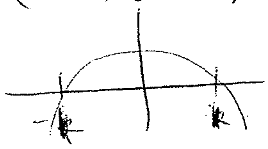
$$b^2 - 4ac > 0$$

$$4 - 4(k)(k) > 0.$$

$$4 - 4k^2 > 0.$$

$$1 - k^2 > 0$$

$$(1-k)(1+k) > 0$$



$$\boxed{-1 < k < 1}$$

e) (i) $\sin\theta = \frac{16}{AQ}$.

$$AQ = \frac{16}{\sin\theta}.$$

$\angle BQN = \theta$ (corresponding angles on parallel lines)

$$\therefore \cos\theta = \frac{2}{BQ}$$

$$BQ = \frac{2}{\cos\theta}$$

$$AB = AQ + BQ$$

$$\therefore AB = \frac{16}{\sin\theta} + \frac{2}{\cos\theta}$$

(ii) $AB = 16(\sin\theta)^{-1} + 2(\cos\theta)^{-1}$

$$(AB)' = -16(\sin\theta)^{-2} \cdot \cos\theta - 2(\cos\theta)^{-2} \cdot -\sin\theta$$

$$= \frac{-16\cos\theta}{\sin^2\theta} + \frac{2\sin\theta}{\cos^2\theta} \qquad 1$$

Stat Pts : $(AB)' = 0$.

$$\frac{-16\cos\theta}{\sin^2\theta} + \frac{2\sin\theta}{\cos^2\theta} = 0.$$

$$\frac{-16\cos^3\theta + 2\sin^3\theta}{\sin^2\theta\cos^2\theta} = 0.$$

$$-2(8\cos^3\theta - \sin^3\theta) = 0.$$

$$8\cos^3\theta - \sin^3\theta = 0 \qquad 1.$$

$$(2\cos\theta - \sin\theta)(4\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta) = 0$$

$$2\cos\theta - \sin\theta = 0.$$

$$2\cos\theta = \sin\theta.$$

$$2 = \tan\theta. \qquad 1$$

Test $\tan\theta = 2$

θ	1	$\tan^{-1}2$ (1.1)	2
$(AB)'$	-6	0	19

1

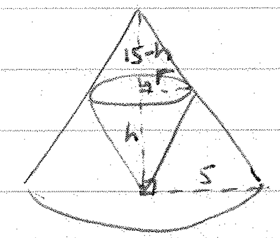
$\therefore$ minimum when $\tan\theta = 2$.

16. a) (i) $\frac{15-h}{r} = \frac{15}{5}$ (corresp. sides of similar triangles in proportion)

$\frac{15-h}{r} = 3$

$15-h = 3r$

$h = 15-3r$



(ii) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (15-3r)$

$= 5\pi r^2 - \pi r^3$

$V' = 10\pi r - 3\pi r^2$

$V'' = 10\pi - 6\pi r$

Stat Pts : $V' = 0$

$10\pi r - 3\pi r^2 = 0$

$\pi r(10-3r) = 0$

$r = 0,\ r = \frac{10}{3}$

But $r > 0$

Test $r = \frac{10}{3}$

$V'' = 10\pi - 6\pi r$

$= 10\pi - 6\pi\left(\frac{10}{3}\right)$

< 0

$\therefore$ max at $r = \frac{10}{3}$

and $h = 15 - 3\left(\frac{10}{3}\right)$

$\underline{h = 5}$

16 b) (i) $m_{PQ} = \dfrac{2-0}{e-1}$

$= \dfrac{2}{e-1}$ 1

(ii) $y = \log_e x^2 = 2\log_e x$

$\dfrac{dy}{dx} = 2 \times \dfrac{1}{x} = \dfrac{2}{x}$

Now $m = m_{PQ}$

$\therefore \dfrac{2}{x} = \dfrac{2}{e-1}$ 1

$2(e-1) = 2x$

$x = e-1.$

$\therefore t = e-1.$ 1

(iii) $R(e-1, 2\ln(e-1))$

$d_{PQ} = \sqrt{(e-1)^2 + 2^2}$

$= \sqrt{e^2-2e+1+4}$

$= \sqrt{e^2-2e+5}$

Equ'n$_{PQ}$: $y - 0 = \dfrac{2}{e-1}(x-1)$

$y = \dfrac{2}{e-1}(x-1)$

$(e-1)y = 2x-2.$

1 $2x-(e-1)y-2=0.$

Perp dist $= \dfrac{|2(e-1)-(e-1)2\ln(e-1)-2|}{\sqrt{4+(e-1)^2}}$

$= \dfrac{|2e-2-2(e-1)\ln(e-1)-2|}{\sqrt{4+e^2-2e+1}}$

$= \dfrac{|2e-4-2(e-1)\ln(e-1)|}{\sqrt{5+e^2-2e}}$

$A_{\Delta PQR} = \dfrac{1}{2} \times \cancel{\sqrt{e^2-2e+5}} \times \dfrac{|2e-4-2(e-1)\ln(e-1)|}{\cancel{\sqrt{e^2-2e+5}}}$

$= \dfrac{2e-4-2(e-1)\ln(e-1)}{2}$

1 $= [e-2-(e-1)\ln(e-1)]\,u^2$

16 (c) (i) $y = \dfrac{\log_e x}{x^2}$

$$\frac{dy}{dx} = \frac{x^2 \cdot \frac{1}{x} - \log_e x \cdot 2x}{x^4}$$

$$= \frac{x - 2x\ln x}{x^4} = \frac{1 - 2\ln x}{x^3} \qquad 1$$

Stat Pts: $\dfrac{dy}{dx} = 0$

$$\frac{x - 2x\ln x}{x^4} = 0 \qquad 1 \qquad \frac{1-2\ln x}{x^3} = 0$$

$$x(1 - 2\ln x) = 0. \qquad 1 - 2\ln x = 0$$

$$x = 0, \quad 2\ln x = 1$$

$$\ln x = \frac{1}{2}$$

$$e^{1/2} = x$$

$$y = \frac{\ln e^{1/2}}{(e^{1/2})^2}$$

$$y = \frac{\frac{1}{2}}{e^1}$$

$$= \frac{1}{2e}$$

$$\therefore P\left(e^{\frac{1}{2}}, \frac{1}{2e}\right) \qquad 1$$

(ii) As $x \to \infty$ $y \to 0$

$\frac{\log_e x}{x^2} = h$ has two values when

$h > 0$ but $h < \frac{1}{2e}$

i.e. $0 < h < \frac{1}{2e}$

(iii) $k = \frac{\ln x}{x^3}$

k max/min when $k' = 0$

$$k' = \frac{\frac{1}{x} \cdot x^3 - 3x^2 \cdot \ln x}{x^6}$$

$$= \frac{1 - 3\ln x}{x^4}$$

$$= x^{-4} - 3x^{-4} \ln x$$

$$k'' = -4x^{-5} - (-12x^{-5} \ln x + \tfrac{1}{x} \cdot 3x^{-4})$$

$$= \frac{-4 + 12\ln x - 3}{x^5}$$

$k' = 0$

$1 - 3\ln x = 0$

$3\ln x = 1$

$x = e^{1/3}$

$$k'' = \frac{-4 + 12 \times \frac{1}{3} - 3}{e^{5/3}}$$

< 0

$$k = \frac{\ln e^{1/3}}{(e^{1/3})^3}$$

$$= \frac{1}{3e}$$

$\frac{\ln x}{x^2} = kx$ has two values when

$k > 0$ but $k < \frac{1}{3e}$

i.e.

$0 < k < \frac{1}{3e}$